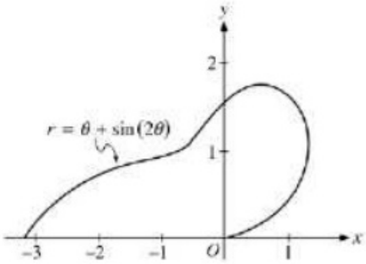


2005 BC2 (Calculator)

The curve above is drawn in xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

a. Find the slope of the curve at the point $\theta = \frac{\pi}{2}$.

c. Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

d. For $\frac{\pi}{2} < \theta \le \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

e. Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

Half Circle

$A = \frac{1}{2}\pi r^2$

$= \left(\frac{\pi}{2\pi}\right)\pi r^2$

One Quarter Circle

$A = \frac{1}{4}\pi r^2$

$A = \left(\frac{\frac{\pi}{2}}{2\pi}\right)\pi r^2$

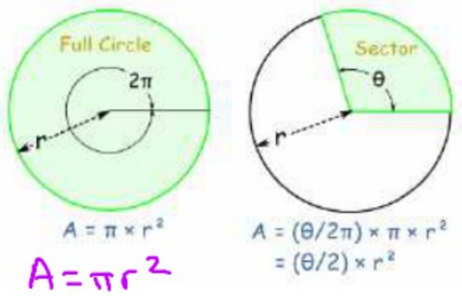
Polar Area

Area of a Sector

You can work out the Area of a Sector by comparing its angle to the angle of a full circle.

Note: we are using radians for the angles.

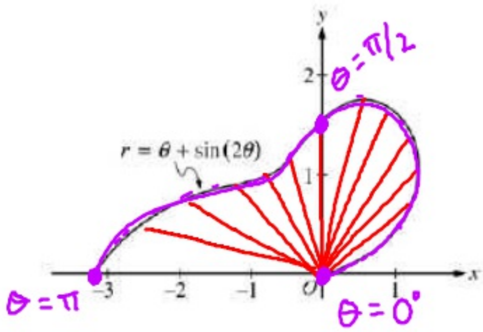
How much of the circle



$A = \pi r^2$

$A = \left(\frac{\theta}{2\pi}\right)\pi r^2$

$A = \left(\frac{\theta}{2}\right) r^2$



$A = \int \frac{\theta}{2} r^2 \, d\theta$

$$\boxed{A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 \, d\theta}$$

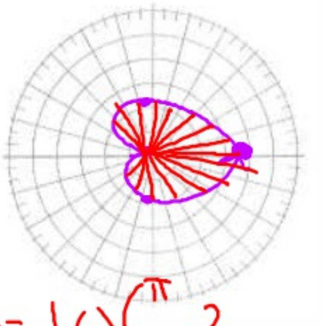
The curve above is drawn in xy-plane and is described by the equation in polar coordinates $\boxed{r = \theta + \sin(2\theta)}$ for $\boxed{0 \le \theta \le \pi}$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

b. Find the area bounded by the curve and the x-axis

$$A = \frac{1}{2}\int_0^{\pi} (\theta + \sin(2\theta))^2 \, d\theta =$$

$\theta = 0$
$r = 2 + 2\cos(0)$
$r = 4$

Find the area inside the polar curve $r = 2(1+\cos\theta)$.



$r = 2 + 2\cos\theta$

$A = \frac{1}{2}\int_0^{2\pi} (2+2\cos\theta)^2$

$A = \frac{1}{2}\int_0^{2\pi} (2+2\cos\theta)(2+2\cos\theta)$

$A = \frac{1}{2}\int_0^{2\pi} 4 + 8\cos\theta + 4\cos^2\theta$

$A = \frac{1}{2}(2)\int_0^{\pi} r^2$

$\frac{0}{2} = \frac{2\sin(2\theta)}{2}$

$0 = \sin(2\theta)$

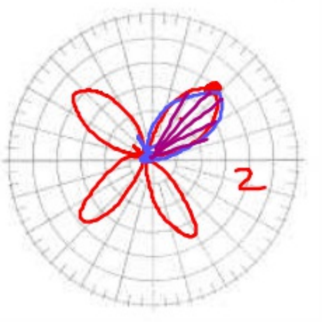
$\frac{2\theta}{2} = \frac{0}{2}, \frac{\pi}{2}, \frac{2\pi}{2}$

$\theta = 0, \frac{\pi}{2}, \pi$

$A = \frac{1}{2}\cdot\frac{1}{4}\int_0^{2\pi} r^2$

$A = \int_{\frac{\pi}{4}}^{3\pi}$

Find the area inside the polar curve $r = 2\sin(2\theta)$.

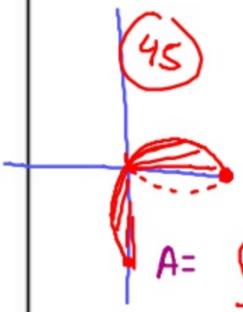


$\theta = 0$
$r = 0$

$A = \frac{1}{2}(4)\int_0^{\frac{\pi}{2}} (2\sin(2\theta))^2$

$A = \frac{1}{2}\int_0^{2\pi} (2\sin(2\theta))^2$

$A = \frac{1}{2}(8)\int_0^{\pi/4} (2\sin(2\theta))^2$



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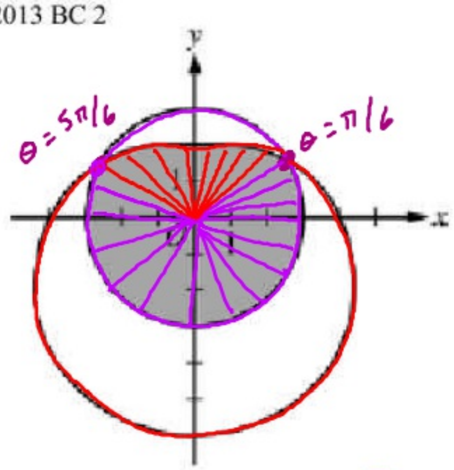
$A = \int_0^{\pi/2} r^2$

$$A = \frac{1}{2}(2)\int_{\frac{5\pi}{6}}^{3\pi/2} (3)^2\, d\theta$$

2013 BC 2



$$A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (4-2\sin\theta)^2\, d\theta$$

$$A = \frac{1}{2}(8)\int_0^{\pi/6} (3)^2$$

OR

$$A = \frac{1}{2}\int_{\frac{5\pi}{6}}^{13\pi/6} (3)^2$$

The graphs of the polar curves $\boxed{r = 3}$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

a. Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

b. A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

$$x = r\cos\theta$$

$$-1 = \underbrace{(4 - 2\sin t^2)\cos(t^2)}_{y_2}$$

y_1

c. For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.